Regular Pumping Lemma Jay Bagga

1 Regular Pumping Lemma

Regular pumping lemma is a powerful tool that helps you show that certain languages are not regular. The statement of the regular pumping lemma is as follows:

If L is a regular language, then there exists an integer m such that given a word w in L, with $|w| \ge m$, w can be decomposed as w = xyz, with the following properties:

- 1. $|xy| \leq m$
- 2. |y| > 0, and
- 3. for each $i \ge 0, xy^i z \in L$.

To use the pumping lemma to show that a given language L is not regular, we proceed by assuming that L is regular so that L satisfies the pumping lemma. Then we select an appropriate w in L and show that w cannot be decomposed as prescribed by the pumping lemma. This contradiction shows that our assumption must be wrong and hence L is not regular.

We will use the language $L = \{w \in \{a, b\}^* : n_a(w) < n_b(w)\}.$

Thus L is the language of those words in $\{a, b\}^*$ that have fewer as than bs.

We first give a direct argument to show that L is not regular. Then we use JFLAP to practice showing the same result.

2 Direct Proof

Assume that L is regular. Then L satisfies the pumping lemma and hence there is an integer m such that the three properties above are satisfied. Select $w = a^m b^{m+1}$. Then $w \in L$ and $|w| \geq m$. By pumping lemma, $a^m b^{m+1} = w = xyz$. By property 1. of the pumping lemma, y must be a string of as. Suppose $y = a^k$. By property 2. of the pumping lemma, k > 0. Then the string $xy^2z = xyyz = a^{m+k}b^{m+1}$ is not in L. This is a contradiction to property 3. Hence L must be nonregular.

3 Proof with JFLAP

JFLAP provides a proof by letting you play a game against the computer. The idea of the game is to win by providing a word w for which pumping lemma fails. Run JFLAP, select

		JFLAP : <untilled9></untilled9>	
JFLAP : <untiled9></untiled9>	~	File Help	
Select a Pumping Lemma	~	Select a Rumping Lomma Rumping Lomma	
First choose who makes the first move		Select a rumping Lemma - Pumping Lemma	
You go first Computer goes first		$L = \{w \in \{a, b\}^* : n_a(w) < n_b(w)\}$ Regular Pumping Lemma	
Then select a lemma.		Objective: Find a valid partition that can be pumped.	
$L = \langle a^n b^n : n \ge 0 \rangle$	Select	Clear All Explain	
$L = \{w \in \{a,b\}^*: n_a(w) < n_b(w)\}$	Select		
$L = (ww^R : w \in \{a, b\}^*)$	Select	1. Please select a value for m in Box 1 and press "Enter".	
$L = \{(ab)^n a^k : n > k, k \succeq 0\}$	Select		
$L = \{a^n b^k c^{n+k} : n \ge 0, k \ge 0\}$	Select		
$L = \{a^n b^l a^k : n > 5, l > 3, k \le l\}$	Select		
$L = \{a^n : n \text{ is even}\}$	Select		
$L = (a^n b^k : n \text{ is odd or } k \text{ is even.})$	Select		
$L = \langle bba(ba)^n a^{n-1} \rangle$	Select		
$L = \{b^S w : w \in \{a, b\}^*, 2n_a(w) = 3n_b(w)\}$	Select		
$L=\{b^5w:w\in\{a,b\}^*,(2n_a(w)+5n_b(w))\bmod 3=0\}$	Select		
$L = \{ b^k (ab)^n (ba)^n : k \ge 4, n = 1, 2 \}$	Select		
$L = \{(ab)^{2n}: n = 1, 2,\}$	Select		

Language Selection

First Step of Proof

Figure 1: Step 1

Regular Pumping Lemma, select "You go first" and then select the language L as above from the list provided. See Figure 1. Click on "Explain" button to see a description of why L is nonregular. Since you go first, JFLAP asks you to enter a number m. Enter 4. See Figure 2.

JFLAP : <untitled9></untitled9>	JFLAP : <untitled9></untitled9>
File Help	X File Help X
Select a Pumping Lemma Pumping Lemma	Select a Pumping Lemma Pumping Lemma
$L = \{w \in \{a, b\}^{\circ} : n_{\mu}(w) < n_{\mu}(w)\}$ Regular Pumping Lemma	$L = \{w \in \{a, b\}^* : n_{-}(w) < n_{+}(w)\} \text{ Regular Pumping Lemma}$
Objective: Find a valid partition that can be pumped.	Objective: Find a valid partition that can be pumped.
Clear All Explain would be a multiple of "a". For any $i > 1$, $n_a \ge n_b$, giving a string which is not in the language. Thus, the language is not regular.	$\begin{array}{c c c c c c c c c c c c c c c c c c c $
1. Please select a value for m in Box 1 and press "Enter".	1. Please select a value for m in Box 1 and press "Enter".
4	4
2. I have selected w such that w >= m. It is displayed in Box 2. aaaabbbbb	2. I have selected w such that w >= m. It is displayed in Box 2. aaaabbbbb
3. Select decomposition of w into xyz. x: x : 0	3. Select decomposition of w into xyz. x: a x : 1
y. Iyi.o Set x	xyz Set xyz
z: z :	z: aabbbbb z : 7
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Figure 2: Step 2

At this point JFLAP gives you the word $w = a^4b^5$ and asks you to provide a decomposition of w. Use the sliders to select x and y. Because of properties 1. and 2. of the pumping lemma, y must be one of a, aa, aaa, aaaa. Select y = aa and then select x = a. z is then selected for you. If x and y are selected appropriately, then the button "Select xyz" can be clicked. Click it. JFLAP then presents you with i = 2 and shows that the string xyyz is not in L. You can animate that proof. See Figure 3. Repeat this entire process for some other values of m.

We now do this again but this time we'll let the "Computer go first." Suppose it selects 15 as in Figure 4. You are asked to select a w. Make sure that w is in L and $|w| \ge 15$. What is an appropriate w? See Figure 4. With the right choice of w and then an i, you win. Try choosing different ws. Repeat the entire process two or three times.

Cle	ar All	Explai	n My I: X	Attempts = a; Y =	aa; Z = ab	obbb; I = 2	; Failed		
1. Plea	ase select	a value f	or m in B	ox 1 and	press *En	ter".			
4									
aaaabb 3. Sele x: a	obbb	position	of w into	xyz.	is unspin		x :	1	
_	-0-								_
y: aa							y :	2	Set xyz
z: abł	bbbb	, (z :	6	-
a	a	a	a	b	b	b	b	b	
4. I ha i: 2	ve select	ed i to giv pum	ve a contr ped strin	adition. I g: aaaaaa	t is displa bbbbb	iyed in Bo	x 4.		
5. Anii	mation X X W = A	y aa a	z bbbbb)					

Figure 3: Step 3

	Select	a Pumping Lemma	Pumping Lemma	
	$I = I_W \subseteq I_R K$	1 ⁸ · n (w) < n (w	Begular Pumping Lemma	
Objective: Prevent	$L = \{w \in \{u, v\}$	om finding a valu	1 partition	
Clear All	Explain Ny /	\ttempts: = aaaaaaaaaaaaaa; \	i = a; Z = bbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbb	2; Won
1. I have selected 15	a value for m, di	splayed below.		
2. Please enter a p	possible value fo	r w and press *Ent	er".	
aaaaaaaaaaaaaaaaa	bbbbbbbbbbbbbbb	bbb		
-4. Please enter a p	X = aaaaaaa possible value fo	iaaaaaaa; Y = a; r i and press "Ente umped string: aaa	Z = bbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbb	bbb
F. Automation				
5. Animation	x	У	z	

Figure 4: Computer goes first

4 References

- 1. Introduction to the Theory of Computation (Third Edition), Michael Sipser. Cengage Learning. 2013.
- 2. JFLAP An Interactive Formal Languages and Automata Package, Susan H. Rodger and Thomas W Finley. Jones and Bartlett Publishers. 2006